VIEW FACTORS IN A SYSTEM OF PARALLEL CONTACTING CYLINDERS

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Equations and the results of calculations of the diffuse-radiation view factors are given for a system of three cylinders.

The problem is solved in connection with the development of a method for calculating the cooling of steel rolls after rolling.

1. The geometry of the system is shown in Fig. 1. The medium is diathermic. Using contour integration [1], we obtain an equation for the view factor (VF) from an elementary area dS_M on the surface S_P:

$$\varphi_{dM-P}(\bar{d}, \bar{l}) = (k/2\pi)\int_{\varphi_1}^{\varphi_2} [f(\varphi, \psi_2) - f(\varphi, \psi_1) + Q(\varphi, z_2) - Q(\varphi, z_1)]\, d\varphi. \quad (1)$$

Here

$$k = 2\cdot 3/\pi;\ \varphi_1 = 2\pi/3;\ \varphi_2 = \pi;\ f(\varphi, \psi_i) = (\sin(\varphi - \psi_i) -$$

$$- 2\sin\varphi)\,\mathrm{arctg}\left[\frac{z_2/A - z_1/A}{1 + (z_2/A)(z_1/A)}\right]\Big/ A;\ Q(\varphi, z_i) = -(z_i/b)\{\cos(\varphi+\theta)\times$$

$$\times \ln(a + b\sin u) + \sin(\varphi+\theta)[u - 2aB\,\mathrm{arctg}((a\,\mathrm{tg}(u/2) + b)B)]\}\Big|_{u_1}^{u_2},$$

$$\text{где}\ \ A = (2(3 - \cos(\psi_i - \varphi) + 2(\cos\varphi - \cos\psi_i)))^{0,5};$$

$$z_1 = \bar{d};\ z_2 = \bar{l} + \bar{d};\ \theta = \mathrm{arctg}((2 + \cos\varphi)/\sin\varphi);$$

$$a = z_i^2 + 2(3 + 2\cos\varphi);\ b = -2(5 + 4\cos\varphi)^{0,5};$$

$$B = (a^2 - b^2)^{-0,5};\ u_i = \psi_i + \theta;\ i = \overline{1, 2}.$$

The angles ψ_1 and ψ_2 depend on φ. It is readily determined (Fig. 2) that

$$\psi_1 = \arccos\left[\frac{(\mathbf{OO}_2, \cdot\, \mathbf{CO}_2)}{|\mathbf{OO}_2|\,|\mathbf{CO}_2|}\right]; \quad (2)$$

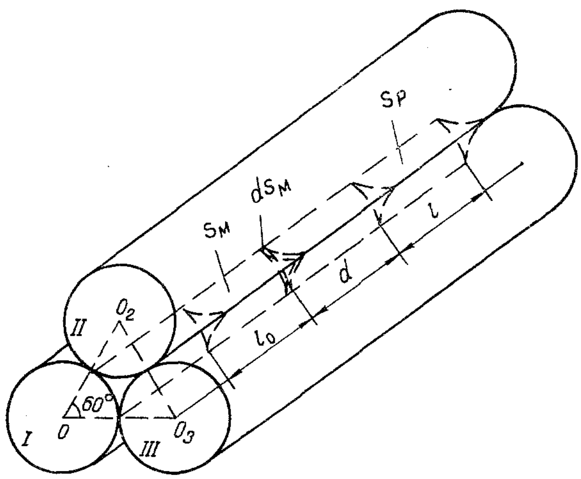

Fig. 1. System of three contacting cylinders.

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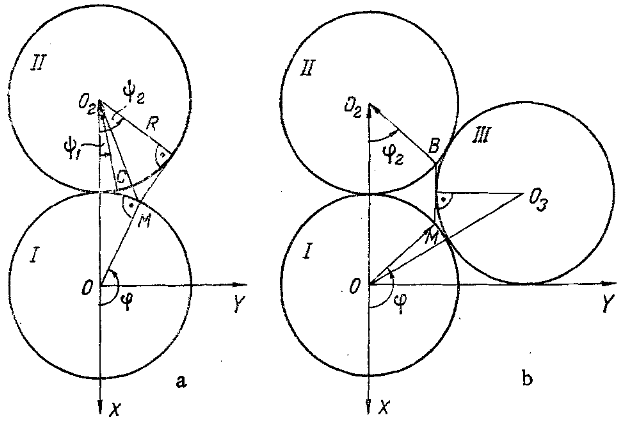

Fig. 2. Diagrams illustrating the determination of the angles ψ_1 and ψ_2. a) $\psi_2 \geq 5\pi/6$; b) $\psi_2 < 5\pi/6$.

TABLE 1. Values of φ_{dM-P} as a Function of d/R and ℓ/R (Fig. 1)

d/R	l/R								
	0,01	0,02	0,05	0,10	0,15	0,30	1	10	100
0,0	0,154	0,206	0,290	0,361	0,400	0,456	0,496	0,500	0,500
0,02	0,35—1	0,62—1	0,118	0,173	0,206	0,254	0,290	0,294	0,294
0,05	0,18—1	0,34—1	0,70—1	0,110	0,135	0,175	0,206	0,209	0,209
0,10	0,96—2	0,18—1	0,40—1	0,65—1	0,83—1	0,111	0,136	0,139	0,139
0,15	0,60—2	0,11—1	0,25—1	0,43—1	0,55—1	0,77—1	0,97—1	0,99—1	0,99—1
0,20	0,40—2	0,77—2	0,17—1	0,30—1	0,39—1	0,56—1	0,72—1	0,74—1	0,74—1
0,30	0,21—2	0,40—2	0,92—2	0,16—1	0,21—1	0,31—1	0,42—1	0,44—1	0,44—1
0,40	0,12—2	0,23—2	0,53—2	0,94—2	0,13—1	0,19—1	0,26—1	0,28—1	0,28—1
0,70	0,29—3	0,57—3	0,13—2	0,24—2	0,33—2	0,53—2	0,82—2	0,91—2	0,91—2
1,0	0,97—4	0,19—3	0,45—3	0,84—3	0,12—2	0,19—2	0,32—2	0,38—2	0,38—2

$$\psi_2 = \begin{cases} \arccos\left[\dfrac{(\mathbf{OO_2},\ \mathbf{MO_2})}{|\mathbf{OO_2}|\,|\mathbf{MO_2}|}\right] + \arccos\dfrac{R}{|\mathbf{MO_2}|}, & \varphi \geqslant 5\pi/6 \text{ (Fig. 2a)}; \\ \arccos\left[\dfrac{(\mathbf{OO_2},\ \mathbf{BO_2})}{|\mathbf{OO_2}|\,|\mathbf{BO_2}|}\right], & \varphi < 5\pi/6 \text{ (Fig. 2b)}. \end{cases} \tag{3}$$

The projections of the vectors in Eqs. (2) and (3) onto the x and y axes are easily determined from geometrical considerations, and we shall not discuss this problem. Table 1 gives the values of $\varphi_{dM-P}(\bar{d}, \bar{\ell})$ calculated according to Eqs. (1)-(3).

2. It is obvious that the equation

$$\varphi_{M-P}(\bar{l}_0, \bar{d}, \bar{l}) = (1/\bar{l}_0)\int_0^{\bar{l}_0} \varphi_{dM-P}(z+\bar{d}, \bar{l})\,dz,$$

in which φ_{dM-P} is calculated according to Eqs. (1)-(3), determines the VF between the surfaces S_M and S_P (see Fig. 1).

3. Suppose that the cylinder III is eliminated. We set $k = 1/\pi$, $\varphi_1 = \pi/2$, and $\varphi_2 = \pi$ in Eq. (1); we calculate ψ_2 according to the first equation (3). We obtain the VF $\varphi_{dM-P}(\bar{d}, \bar{\ell})$ between an elementary ring and a cylinder of length ℓ (Fig. 3a).

The equation

$$\varphi'_{M-P}(\bar{l}) = (2/\bar{l})\int_0^{\bar{l}/2} [\varphi'_{dM-P}(0,\ \bar{l}-z) + \varphi'_{dM-P}(0,\ z)]\,dz \tag{4}$$

obviously determines the VF between two contacting cylinders of length ℓ (Fig. 3b).

A comparison of the calculations according to Eq. (4) with the results of [2] indicates good agreement between them (Table 2).

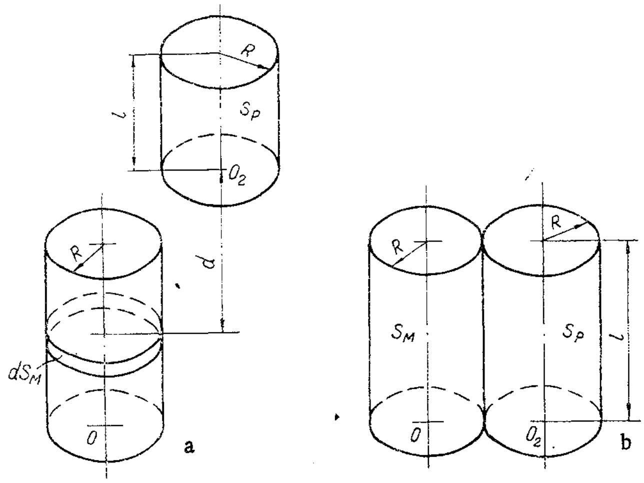

Fig. 3. Diagrams illustrating the determination of the view factors in a system of two contacting cylinders. a) VF φ_{dM-P} between an elementary ring and a cylinder of length ℓ; b) VF φ_{M-P} between two cylinders.

TABLE 2. Comparison of the Results of Calculations of φ'_{M-P} According to Eq. (4) with the Results of [2]

l/R	Calc. per Eq. (4)	Results of [2]	l/R	Calc. per Eq. (4)	Results of [2]
0,5	0,1418	0,1440	10	0,1791	0,1813
1	0,1584	0,1603	50	0,1811	0,1834
5	0,1765	0,1773			

NOTATION

R, tube radius; ℓ_0, ℓ, lengths of sections S_M and S_P; d, distance between same sections; $\bar{\ell}_0 = \ell_0/R$; $\bar{\ell} = \ell/R$; $\bar{d} = d/R$; φ_{dM-P}, φ'_{dM-P}, view factor from element dS_M to section S_P; φ_{M-P}, φ'_{M-P}, the same from section S_P.

LITERATURE CITED

1. E. M. Sparrow and R. D. Cess, Radiation Heat Transfer, Brooks Publ. Co., Belmont, CA (1966).
2. N. H. Juul, Trans. ASME Ser. E: J. Heat Transfer, 104, No. 2, 384 (1982).